



RESUME OF MATHEMATICS

1. STANDARD OF THE PAPERS

The Chief Examiners for Mathematics (Core) 2 and Mathematics (Elective) 2 agreed that the standard of their respective papers compared favourably with that of the previous years.

The standard of the paper compared favourably with that of the previous years.

Candidates' performance was, however, lower than that of the previous year.

2. A SUMMARY OF CANDIDATES' STRENGTHS

The following strengths of candidates were highlighted:

- (i) simplifying a given expression into standard form;
- (ii) simplification of surds;
- (iii) finding the base of a given equation;
- (iv) completing table of values of a trigonometric relation and drawing the graph of the relation using a given scale and interval;
- (v) completing table of values for multiplication in modulo 11 and using it to solve related problems;
- (vi) using a given Venn diagram to solve problems involving three intersecting sets.

3. A SUMMARY OF CANDIDATES' WEAKNESSES

Candidates weaknesses were observed in:

- (i) lack of ability to translate word problem into mathematical statement;
- (ii) lack of understanding of basic concepts of plane geometry and circle theorem;
- (iii) inability to solve and evaluate given functions;
- (iv) inability to show the commutativity of a binary operation.

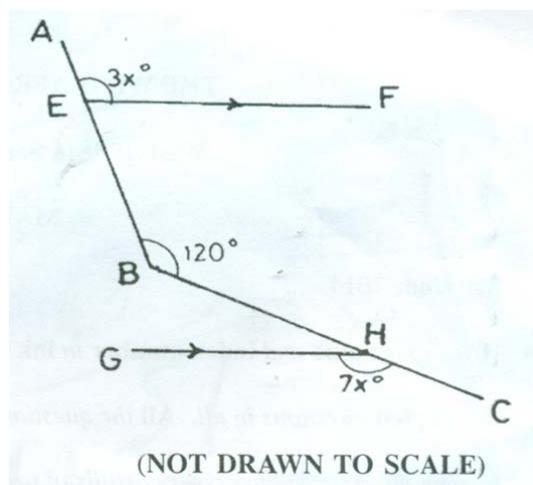
4. SUGGESTED REMEDIES

- (1) Teachers should prepare the candidates very well by giving them more exercises particularly on the above weaknesses and guide them to work.
- (2) Candidates should do a thorough revision before the examinations.
- (3) Candidates should read the questions very well before answering them and ensure that the rubrics are followed.

5. DETAILED COMMENTS

Question 1

- (a) Without using tables or calculator, simplify: $\frac{0.6 \times 32 \times 0.004}{1.2 \times 0.008 \times 0.16}$ leaving the answer in standard form (*scientific notation*).
- (b) In the diagram, \overline{EF} is parallel to \overline{GH} . If $\angle AEF = 3x^\circ$, $\angle ABC = 120^\circ$ and $\angle CHG = 7x^\circ$, find the value of $\angle GHB$.



- (a) The elimination of the decimal to facilitate simplification was done by most candidates.

However some of the candidates ignored the concept of standard form and used calculators in simplifying the expression but finally they left their answers in standard form.

- (b) Most candidates did not draw the diagram in order to analyse the question. They were unable to visualize and extend the parallel lines EF and GH in the diagram to facilitate the application of the geometrical theorems to obtain the required solution.

Question 2

- (a) **Simplify $3\sqrt{75} - \sqrt{12} + \sqrt{108}$, leaving the answer in surd form (radicals).**
- (b) **If $124_n = 232_{\text{five}}$, find n .**
- (a) Most candidates expressed each term in the given surds as a product of two numbers with one being a perfect square. They managed to simplify the expression and left their answer in a surd form. However, some could not express the terms of the surds as a product of a perfect square and another number but rather simplified the expression as given.
- (b) Most candidates were able to convert the terms on both sides of the equation into a base ten numeral, and obtained the required quadratic equation as $n^2 + 2n - 63 = 0$. They solved the equation and obtained two values of n but some of them could not specifically conclude that the base is the positive value of n .

Question 3

- (a) **Solve the simultaneous equations:**

$$\frac{1}{x} + \frac{1}{y} = 5$$

$$\frac{1}{y} - \frac{1}{x} = 1$$

- (b) **A man drives from Ibadan to Oyo, a distance of 48 km, in 45 minutes. If he drives at 72 km/h where the surface is good and 48 km/h where it is bad, find the number of kilometers of good surface.**

- (a) The few candidates who were able to identify the same term which had different operational signs in the simultaneous equation solved the equation with ease. They used elimination method in solving for the x and y values. Others attempted the question using substitution method but it resulted in complex equation which they could not solve.
- (b) Most candidates found it difficult to analyse the question in order to form the required equations. Candidates were expected to state the formula for speed and then find the total time spent on both good and bad surfaces.

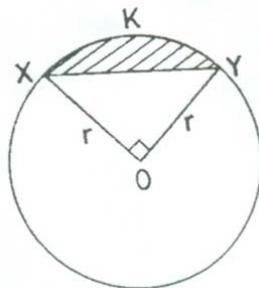
From the analysis they were expected to obtain the time spent on the good surface as

$T = \frac{x}{72}$ hrs where x is the distance covered and $T = \frac{48-x}{48}$ hrs as time spent on the bad surface. To obtain the number of kilometers of good surface, candidates were expected to add the time spent on each of the surfaces and equate it to the total time for the journey.

Candidates inability to write the relevant equations give an indication that they did not understand the question.

Question 4

- (a) In the diagram, O is the centre of the circle radius r cm
 Y
 and $\angle XOY = 90^\circ$. If the area of the shaded part is 504cm^2 . Calculate the value of r , [Take $\pi = \frac{22}{7}$].



- (b) Two isosceles triangles PQR and PQS are drawn on opposite sides of a common base PQ . If $\angle PQR = 66^\circ$ and $\angle PSQ = 109^\circ$, calculate the value of $\angle RQS$.

- (a) Most candidates identified the shaded part of the given circle as the difference of the areas of the sector KXOY and the isosceles triangle XOY. They substituted the given parameters into the respective formulae of the areas correctly and equated it to the given area of the shaded portion. They manipulated the equation

and quite a number of them were able to find the radius (r) of the circle. However, others could not identify the shaded portion of the given circle as the difference of the areas of the sector KXOY and the isosceles triangle XOY rather they identified it as a segment but they could not relate it to the isosceles triangle.

- (b) Most candidates drew two isosceles triangles with a common base PQ and indicated the given angles correctly. They applied the concept of equality of base angles of isosceles triangles to find the relevant base angles which they added to 66° to obtain the required angle. Others could not sketch the diagram let alone find the required angle.

Question 5

A building contractor tendered for two independent contracts, X and Y. The probabilities that he will win contract X is 0.5 and not win contract Y is 0.3. What is the probability that he will win:

- (a) **both contracts;**
(b) **exactly one of the contracts;**
(c) **neither of the contracts?**

Most candidates expressed the probability of winning both contracts as the product of the probability of winning each contract and simplified the expression correctly.

Again, most of the candidates demonstrated good understanding of negation in probability by correctly applying it to find the probability of winning **exactly one** and **neither** of the contracts. However, there were computational errors which could have been avoided, if they had taken their time.

Question 6

- (a) If $\frac{3}{2p-\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{4p}+1}$, find p .

- (b) A television set was marked for sale at GH¢ 760.00 in order to make a profit of 20%. The television set was actually sold at a discount of 5%. Calculate, correct to 2 significant figures, the actual percentage profit.
- (a) Most candidates were able to carry out the cross-multiplication and then cleared the fractions by multiplying through by the appropriate least common multiple. This enabled them to solve for the variable (p). However, after the cross-multiplication, some of the candidates could not proceed, while others used a wrong approach to clear the fractions.
- (b) Most candidates could not understand the concept of marked price for sale at a “given percentage profit”, and therefore messed up with the computation of the actual percentage profit. However, a few candidates were able to compute the actual percentage profit correctly.

Question 7

- (a) Copy and complete the table of values for the relation $y = 2 \sin x + 1$.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°
y	1.0				2.7			0.0	-0.7	

- (b) Using scales of 2 cm to 30° on the x-axis and 2 cm to 1 unit on the y-axis, draw the graph of $y = 2 \sin x + 1$ for $0^\circ \leq x \leq 270^\circ$.
- (c) Use the graph to find the values of x for which $\sin x = \frac{1}{4}$.

- (a) Most candidates copied and completed the given table, plotted the points and joined them to obtain a smooth curve.

Even though most candidates were able to draw the graph they were unable to use the graph to find the values of x for which $\sin x = \frac{1}{4}$. Instead of candidates manipulating $\sin x = \frac{1}{4}$ to obtain $2 \sin x + 1 = 1.5$ which could be compared to the original equation and then deduce that $y = 1.5$, they rather solved $\sin x = \frac{1}{4}$ to obtain $x = 14^\circ$. Some candidates found it difficult to draw the line $y = 1.5$, in order to read off the values of x at the point where the line cuts the curve.

Question 8

- (a) Copy and complete the following table for multiplication modulo 11.

\otimes	1	5	9	10
1	1	5	9	10
5	5			
9	9			
10	10			

Use the table to:

- (i) evaluate $(9 \otimes 5) \otimes (10 \otimes 10)$;
(ii) find the truth set of
(a) $10 \otimes m = 2$;
(β) $n \otimes n = 4$
- (b) When a fraction is reduced to its lowest term, it is equal to $\frac{3}{4}$. The numerator of the fraction when doubled would be 34 greater than the denominator. Find the fraction.
- (a) Most candidates who attempted the question completed the multiplication Modulo 11 table correctly. They showed mastery in solving the related questions. However, a few candidates did not use the curly brackets in presenting their final answer to question a(ii).
- (b) Most candidates could not translate the word problem into the correct mathematical statement, hence their inability to solve the problem. However, few candidates were able to analyse the problem and came out with the required simultaneous equations as $\frac{x}{y} = \frac{3}{4}$ and $2x = y + 34$.

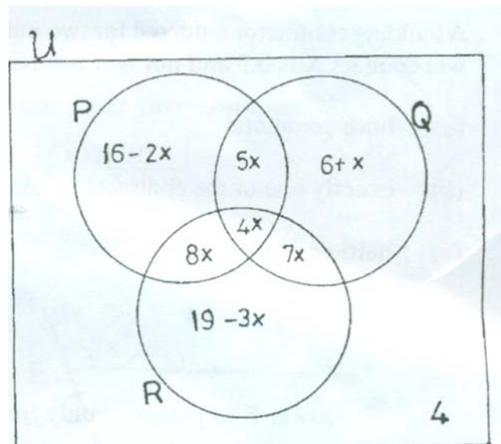
The few candidates who were able to obtain the required equations managed to solve for the values of x and y.

Question 9

- (a) In the Venn diagram, P, Q and R are Subsets of the universal set U. If

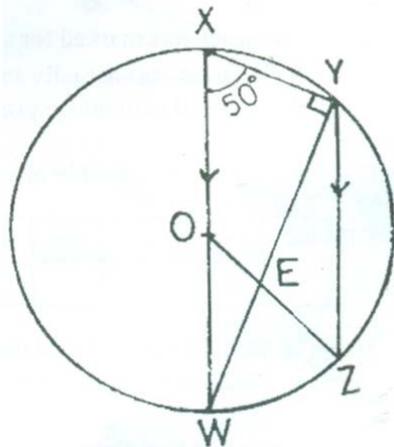
$n(U) = 125$. Find:

- (i) the value of x ;
(ii) $n(P \cup Q \cap R)$;



- (b) In the diagram, O is the centre of the circle. If WX is parallel to YZ and $\angle WXY = 50^\circ$, find the value of:

- (i) $\angle WXZ$
(ii) $\angle YEZ$

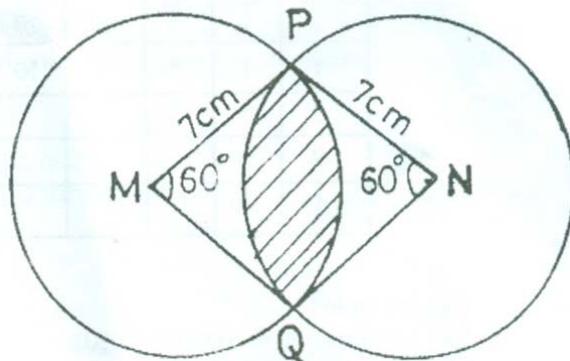


- (a) (i) Most candidates were able to use the information provided in the Venn diagram to write the required equation and solved for the value of x .

- (iii) Most candidates could not identify the regions which corresponds to $n(P \cup Q \cap R')$ hence their inability to find the number of elements in $n(P \cup Q \cap R')$. Even though the question posed a lot of challenge to most of the candidates, some were able to identify the required region and solved the problem.
- (b) Candidates were expected to use the circle theorems to find the values of angles WXZ and YEZ , but most candidates could not recall the relevant circle theorems which would enable them to solve the problem. In view of this most candidates did not attempt the question. However, some of the candidates who made the attempt applied the concept of circle theorems appropriately to find the values of angles WXY and YEZ .

Question 10

- (a) Solve: $(x - 2)(x - 3) = 12$.
- (b) In the diagram, M and N are the centres of the two circles of equal radii 7 cm. The circles intersect at P and Q . If $\angle PMQ = \angle PNQ = 60^\circ$, calculate correct to the nearest whole number, the area of the shaded portion.
- [Take $\pi = \frac{22}{7}$]



- (a) Expansion of $(x-2)(x-3)$ was correctly done by most candidates. Simplification and re-arrangement for the required equation eluded a good number of them. However, candidates who were able to obtain the correct quadratic equation, had the values of x correctly.

- (b) A few of the candidates who answered this question were able to identify the shaded region as twice the difference between the area of the sector and the triangle. Candidates were able to state and use the formulae for finding the area of a sector and triangle accurately. It was noted that some candidates left their final answers in decimals while others did not use the appropriate units.

Question 11

Scores	1	2	3	4	5	6
Frequency	2	5	13	11	9	10

The table shows the distribution of outcomes when a die is thrown 50 times. Calculate the:

- (a) **mean deviation of the distribution;**
 (b) **probability that a score selected at random is at least a 4.**

- (a) Most candidates were able to find the mean of the data but messed up in finding the mean deviation. Instead of taking the absolute values of the deviations, some candidates did not and therefore had the mean deviation of the distribution wrong. However, a good number of the candidates showed mastery in finding the mean deviation.
- (b) Most candidates interpreted at least 4 correctly by summing up the frequencies starting from the score 4 up to score 6. They went ahead to use the sum of the frequencies of at least 4 to find the probability. The approach was very encouraging.

On the other hand, some of the candidates could not interpret at least 4, and therefore expressed the frequency of score 4 as a fraction of the total frequency. Their approach was entirely wrong.

Question 12

- (a) **Given that $5 \cos (x + 8.5)^\circ - 1 = 0$, $0 \leq x \leq 90^\circ$, calculate, correct to the nearest degree, the value of x .**
- (b) **The bearing of Q from P is 150° and the bearing of P from R is 015° . If Q and R are 24 km and 32 km respectively from P:**
- (i) **represent this information in a diagram;**
 (ii) **calculate the distance between Q and R, correct to two decimal places;**
 (iii) **find the bearing of R from Q, correct to the nearest degree.**

- (a) Most candidates could not simplify the given equation to obtain $x + 8.5^\circ = 78.46^\circ$, let alone solve for the value of x . Others wrongly expanded it as $\cos x + \cos 8.5 = 0.2$, and then messed up with the whole computation.

However, some candidates were able to simplify the given equation correctly and then solved for the value of x .

- (b) Most candidates could not draw a meaningful diagram to represent the given information. This affected the calculation of the distance and the bearing of the given points.

However, candidates who were able to accurately draw the diagram, used the cosine rule to find the distance between the two given points Q and R and then proceeded to find the bearing by using the sine rule.

Question 13

- (a) Two functions f and g are defined by $f: x \rightarrow 2x^2 - 1$ and $g: x \rightarrow 3x + 2$, where x is a real number.

(i) If $f(x - 1) - 7 = 0$, find the values of x .

(ii) Evaluate: $\frac{f(-\frac{1}{2}) \cdot g(3)}{f(4) - g(5)}$.

- (b) An operation $*$ is defined on the set \mathbf{R} , of real numbers, by

$$\mathbf{m * n = \frac{-n}{m^2 + 1}, \text{ where } m, n \in \mathbf{R}.$$

If $-3, -10 \in \mathbf{R}$, show whether or not $*$ is commutative.

- (a)(i) Most candidates could not substitute $(x - 1)$ into the function $f(x) = 2x^2 - 1$ to obtain $2(x-1)^2 - 1 - 7 = 0$. Rather they simplified $f(x - 1) - 7 = 0$ to obtain

$$f = \frac{7}{x-1}, \text{ but this approach was entirely wrong.}$$

Some candidates were able to substitution $(x - 1)$ into the function $f(x)$ and solved for the value of x .

- (ii) Most candidates substituted the given values into the functions correctly but could not accurately evaluate it.

- (b) Few candidates demonstrated the correct usage of the commutative property, by stating that for commutativity $m * n = n * m$

$$\frac{-n}{m^2+1} = \frac{-m}{n^2+1}$$

and then went ahead to show whether or not the * is commutative. Finally they

showed that the operation * is not commutative, since changing the operands changed the results.

MATHEMATICS (ELECTIVE) 2

1. GENERAL COMMENTS

The standard of the paper compared favourably with that of the previous years.

Candidates' performance, however, was slightly lower than that of the previous year.

2. A SUMMARY OF CANDIDATES' STRENGTHS

Candidates strengths were evident in:

- (i) binomial expansions;
- (ii) solving quadratic equations;
- (iii) finding the magnitude of a vector;
- (iv) carrying out Matrix multiplication;
- (v) constructing a frequency and cumulative frequency table;
- (vi) addition and subtraction of vectors;
- (vii) using dot product to find the angle between two vectors.

3. A SUMMARY OF CANDIDATES' WEAKNESSES

Candidates weaknesses were observed in:

- (i) solving logarithm equations;
- (ii) use of deviations to calculate the mean and standard deviation;
- (iii) solving equations involving fractions;
- (iv) the correct use of the quotient and product rule in differentiation;
- (v) the use of algebra in calculating probabilities;

- (vi) resolution of forces in quadrants other than the first;
- (vii) application of the concept of the sequence of numbers in solving everyday problems;
- (viii) the correct use of the concept of momentum.

4. **SUGGESTED REMEDIES**

- (1) Teachers should prepare their candidates very well by giving them more exercises particularly on the above weakness and guide them to work them.
- (2) Candidates should do a thorough revision before the examination.
- (3) Candidates should read the questions very well before answering them, ensuring that all rubrics are observed.

5. **DETAILED COMMENTS**

Question 1

Solve $(\log_2 n)^2 + \log_2 n^3 = 10$.

This was a popular question, however, candidates' performance was poor.

Candidates were expected to substitute any variable, other than n , e.g. x for $\log_2 n$ in the given equation. This would then yield $x^2 + 3x - 10 = 0$ which could then be solved for x , and hence n .

Only a few of the candidates who attempted it were able to eventually solve for n . Most of them ended at solving for x only.

Some of the candidates also simplified $(\log_2 n)^2$ to $2\log_2 n$ which was totally wrong.

Question 2

- (a) Expand $(2x + 1)^4$ in descending powers of x .
- (b) Using the expansion in (a), evaluate $(1.05)^4$ correct to four significant figures.

Candidates' performance in this question was good. In part (a) most of the candidates expanded the binomial expression correctly.

A few of them, however, expanded it in ascending powers of x instead of in the descending powers as was required in the question and got penalized for that.

In part (b), using the expansion in (a) to evaluate $(1.05)^4$ was properly done by most of them. A few, however, could not obtain the value for x , some of them even substituted the 1.05 given as x instead of 0.025.

Question 3

A man started a project with \$1,160.00. If he increases this value with \$40.00 every month until he last invested \$1,760.00, calculate the total amount he invested in the project at the time he invested \$1,760.00.

This question was on the application of an Arithmetic Progression (A.P.) in finding the total amount invested in a project but most of the candidates did not recognize this fact. They went about solving the problem without knowing exactly what was expected of them. They messed up everything.

Candidates were expected to first find the number of years involved in the whole transaction, as

$$Un = a + (n - 1)d$$

$$1760 = 1160 + (n-1)40$$

$$n = 16$$

and then the total amount invested as

$$Sn = \frac{16}{2} + [1160 + 1760]$$

$$= \$ 23,360.00$$

Question 4

If α and β are the roots of $3x^2 + 4x - 3 = 0$, find the equation whose roots are $(\alpha + \frac{1}{\beta})$ and $(\beta + \frac{1}{\alpha})$.

The question was popular and candidates' performance was good.

Most of the candidates who attempted the question were able to state the values of $\alpha + \beta$ and $\alpha\beta$, using the constants in the equation $3x^2 + 4x - 3 = 0$, and then found the equation whose roots were $(\alpha + \frac{1}{\beta})$ and $(\beta + \frac{1}{\alpha})$.

Some of the candidates, however, were unable to find the sum and product of the roots of the new equation in terms of the given equation.

Question 5

Two balls with masses 10 kg and 15 kg roll towards each other with velocities 2 ms^{-1} and 3 ms^{-1} respectively until they collide. After collision, the 10 kg ball moves with a velocity of 1.2 ms^{-1} in the opposite direction. Calculate the:

- (a) change in momentum of the 10 kg ball;**
- (b) velocity of the 15 kg ball after collision.**

Candidates performance in this question was poor.

In part (a) since the two balls were moving towards each other before collision their velocities were negative with respect to each other's. Most of the candidates failed to note this.

Also after collision the 10 kg ball moved in the opposite direction to its initial movement. Hence taking the direction after collision as positive, change in momentum of the 10 kg ball

$$= 10(1.2) - 10(-2)$$

$$= 32 \text{ kg ms}^{-1}$$

In part (b) the concept of the conservation law was required.

$$\text{Hence } (15 \times 3) + 10(-2) = 15V + 10(1.2)$$

$$13 = 15V$$

$$V = \frac{13}{15} \text{ ms}^{-1} \text{ or } 0.867 \text{ ms}^{-1}$$

Most of the candidates took each momentum before collision as positive, which was wrong. They took the two bodies as moving with a common velocity after the collision, which again was wrong.

Question 6

The deviations of six numbers from 12 are: -4, 3, 2, 0, 1 and -3. Calculate to three significant figures, the:

- (a) mean;**
- (b) standard deviation of the numbers.**

Candidates' performance in the question was not encouraging. Candidates were given deviations from the number 12 and were to find the mean and standard deviation of the numbers.

They were expected to sum up the deviations and the squared deviations, and then use the assumed mean formulae to solve the problem.

Most of the candidates did not use the 12 at all when finding the mean.

Question 7

A committee of 3 lawyers and 2 judges is to be formed from 8 lawyers and 6 judges. In how many ways can this be done if

- (a) there are no restrictions;**
- (b) two particular lawyers must not be on the committee;**
- (c) one particular judge must be on the committee?**

The question was on combinations and candidates' performance was quite good.

In part (a) where the committee was to be formed without any restriction candidates had no problem with it.

Similarly in part (b) where two particular lawyers must be on the committee, they did not have much problem with it.

However, in part (c) where one particular judge must be on the committee many of them had difficulty with it.

If one particular judge must be on the committee, then 5 of them are left to choose the remaining one from.

Hence the number of ways in which the committee can be formed

$$= {}^8C_3 \times {}^5C_1$$

$$= 280$$

Question 8

The cosine of the angle between the vectors $x = 3i - j$ and $y = 2i + mj$ is $\frac{\sqrt{5}}{5}$. Find the values of m .

The question was popular. Candidates' performance was average. The question was on the application of the concept of the dot product between two vectors in order to find the value of m .

Most of the candidates who attempted it were able to find the dot product and the magnitudes of the vectors. However, some of the candidates took $\frac{\sqrt{5}}{5}$ as the angle, rather than the cosine of the angle between the vectors as stated in the question. Some also simplified $\sqrt{4 + m^2}$ to $2 + m$ which was entirely wrong. Indeed many of the candidates had difficulty handling the radical signs.

Some of the candidates who got the correct quadratic equation failed to solve it.

Question 9

(a) If $y = \frac{10}{(2x^2+1)^5}$, find $\frac{dy}{dx}$

(b) When the polynomial $f(x) = x^3 + ax^2 + bx + 3$, where a and b are constants is divided by $(x + 2)^2$, the remainder is zero. Find the:

- (i) values of a and b ;
- (ii) zeros of $f(x)$.

Candidates' performance in this question was not encouraging. In part (a) candidates were to differentiate $y = \frac{10}{(2x^2+1)^5}$. This involved the use of the product or quotient rule as the easiest way.

$$y = \frac{10}{(2x^2+1)^5} = 10(2x^2 + 1)^{-5}$$

Using the product rule : $\frac{dy}{dx} = 0 + -50(2x^2 + 1)^{-6} \cdot 4x$

$$= \frac{200x}{(2x^2+1)^6}$$

Most of the candidates had difficulty obtaining the second term in the differentiation. Some of the candidates also either left the final answer as $-200x(2x^2+1)^{-6}$ or worse still as $-50(2x^2+1)^{-6} \cdot 4x$. Although the first answer was a simplified form of the second one, it was still not in the form in which the question was given, and candidates were penalized for that.

In part (b) candidates' performance was very poor. Candidates' major problem was that the polynomial had a repeated factor which candidates took as only one factor and substituting the single root yielded only one equation to be solved for two unknowns. This obviously was not possible. They resorted to all sorts of unaccepted methods to create a second equation and everything broke down.

Question 10

Given that $M = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$, $N = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$ and P satisfies the equation $PM^2 + MN = 2I$, where I is 2 x 2 unit matrix, find

- (a) M^2
 (b) P.

The question was popular and generally candidates' performance was average. In part (a) most of the candidates who attempted the question had no difficulty finding M^2 .

A few of the candidates simply multiplied each element by itself in order to obtain M^2 which was unacceptable.

In part (b) most of the candidates were able to find MN.

Others were able to substitute $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ for P in the equation.

However, after obtaining four equations they did not seem to know how to solve them simultaneously for the elements.

Some of the candidates used the inverse method but only a few of them used it properly. A lot of them did not seem to care about the order and position of the inverse in the equation.

Question 11

- (a) Find the equation of the circle which passes through the points (1, 1), (2, 4) and (3, 2)
- (b) The cost, (C), in Ghana cedis, per kilometer of a telephone cable is given by $C = \frac{120}{x} + 30x$, where x is the cross sectional area of the cable in mm^2 . Find the:
- (i) least cost per kilometer;
 - (ii) cross sectional area for which the cost is least.

The question was not popular and the few candidates who attempted it performed very poorly.

In part (a) candidates were to substitute the coordinates of the given points in the general equation of a circle, $x^2 + y^2 + 2gx + 2fy + c = 0$, and solved the three resulting equations simultaneously for the values of g, f and c .

A few of the candidates did it right but were unable to solve for the three unknowns.

In part (b)(i) the question asked for the least cost per kilometer which required differentiation.

Most of the candidates failed to carry out any differentiation at all and yet were trying to find the least cost. The few who attempted the differentiation were unable to carry it out properly.

In part (b)(ii) since candidates could not carry out the differentiation, finding the cross sectional area for which the cost was least, became a problem.

Question 12

The marks scored by some students in a test are

28	35	41	47	62	70	81
59	60	61	62	70	80	68
67	68	69	70	78	57	66
74	76	77	78	54	64	73
88	90	94	51	64	72	83

- (a) **Construct a grouped frequency table for the data using the intervals 20 – 29, 30 – 39, 40 – 49,**
- (b) **Draw a cumulative frequency curve for the distribution.**
- (c) **If 68 is the pass mark, find the:**
- (i) **percentage of students who passed;**
 - (ii) **ratio of students who passed to those who failed.**

The question was popular and candidates' performance was good. Most of the candidates

were able to construct a grouped frequency table using the given intervals properly, as required in part (a).

In part (b) some of the candidates plotted the cumulative frequencies against class midpoints or lower class boundaries or class limits. These were all unacceptable. Some of the candidates also drew frequency polygons contrary to the demands of the question.

In part (c) a few of the candidates were able to use the graph to find the ratio of students who passed to those who failed.

Question 13

A bag contains 4 red balls and a number of blue balls. All the balls are identical except for colour. Two balls are selected at random from the bag one after the other without replacement. If the probability of selecting a red ball and a blue ball is $\frac{5}{9}$, find the number of blue balls in the bag.

The question was not popular and only a few of the candidates who attempted it performed better.

A solution to the question is presented below:

$$\text{Let the number of blue balls} = x$$

$$\text{The total number of balls} = x + 4$$

$$\begin{aligned} \text{P(Red ball followed by Blue ball)} &= \frac{4}{x+4} \cdot \frac{x}{x+3} \\ &= \frac{4x}{(x+4)(x+3)} \end{aligned}$$

$$\text{P(Red ball and Blue ball)} = \text{P(Red ball followed by Blue ball)}$$

OR

(Blue ball followed by Red ball)

$$\begin{aligned} &= \frac{4x}{(x+4)(x+3)} + \frac{4x}{(x+4)(x+3)} \\ &= \frac{8x}{x^2+7x+12} \end{aligned}$$

$$\frac{8x}{x^2+7x+12} = \frac{5}{9}$$

$$5x^2 - 37x + 60 = 0$$

$$x = \frac{37 \pm \sqrt{37^2 - (4)(5)(60)}}{2(5)}$$

$$x = 5 \text{ or } 2.4$$

$$\text{Number of blue balls} = 5$$

Note: The quadratic equation could also be factorized to $(x - 5)(5x - 12)$

The combination method could also be used.

Question 14

- (a) Vectors $p = 2a - b$ and $q = \frac{1}{2}b + 3a$. If $a = i + j$ and $b = 3i - 4j$, find, correct to the nearest whole number, the angle between p and q .
- (b) A body of mass 0.3 kg is subjected to an impulse which alters its velocity from $(-4i + 6j)ms^{-1}$ to $(12i + 2j) ms^{-1}$. Calculate the change in momentum.

The question was popular and candidates' performance was good.

In part (a) candidates were able to express the vectors p and q in the unit vectors i and j properly.

Finding the magnitudes and the dot product of the two vectors were quite well done. Using the dot product the angle between the two vectors could be found.

However, a few of the candidates could not find the expressions for p and q properly. Angles were also not expressed as whole numbers.

In part (b) the change in momentum was properly found. A few of the candidates, however, did not seem to know what to do.

Question 15

- (a) Find the resultant of the forces (8 N, N 35° E), (9 N, N 0° E) and (10 N, S 50° E).
- (b) Find the additional force that will keep the system in (a) in equilibrium.

Candidates' performance in this question was barely average. Candidates did not have much problem resolving the forces in the 1st quadrant. However, they could not handle the one in the 4th quadrant properly. Candidates seemed to be more familiar with directions given in the bearing form than the cardinal points form.

The error committed in finding the resultant force affected their final score greatly, since it was used in subsequent calculations throughout the question.

In part (b) candidates were able to find the equilibrant once they were able to find the resultant in part (a). However, the final answer could not be right since the error in the resolution persisted through all related calculations.